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Required $\sqrt[3]{43}$. Since $2^3 \times 43 = 344$; and $7^3 = 343$ we have

$\left(\frac{344}{147} + \frac{2}{3}.7\right) \div 2 = 3.503401$; the true root being 3503398.

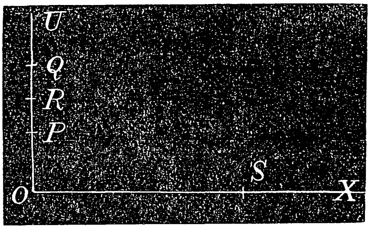
This method is equally applicable to fractional numbers.

Required $\sqrt{5.456789}$. Suppose the root to be $\frac{7}{3}$; then

$\frac{5.456789}{2(\frac{7}{3})} + \frac{1}{2} \times \frac{7}{3} = 2.335977$; true to the last figure inclusive.

FIRST PRINCIPLES OF THE DIFFERENTIAL CALCULUS.

BY S. W. SALMON, MOUNT OLIVE, NEW JERSEY.



LET a point A start from O and move uniformly along the line OX, and let dx represent its rate of motion. At the same instant that A starts from O, let another point B start from P at a distance b from O, and move along the line OU. Let the distances of the contemporaneous positions of A and B from O be represented by x and u respectively, and let the relation between x and u be expressed by the equation

$$u = b + ax^n, \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

in which a and b are constants. When $n = 1$, B will move uniformly; for any other value of n B moves at a rate which either increases as x increases or decreases as x increases. Our problem is to find a general expression for the rate of motion of B, which we will denote by du. Let Q and S, at distances u' and x' from O respectively, be any two contemporaneous positions of A and B. Hence at these points we have

$$u' = b + ax'^n. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (2)$$

Let du' be the rate at which b passes through the point Q; now when du' is known we can find a point in the line OU, R suppose, from which another point C must start simultaneously with A and B so that, moving uniformly along OU at the rate du', it shall coincide with B at the point Q. Let $OR = c$. The time in which B passes over the space $u' - b$, being the same as that in which A passes over the space x', $=$ space $\div$ rate $= x' \div dx$. The space which C passes over in this time $=$ time $\times$ rate

$$= \frac{du'}{dx} x' = u' - C; \; \therefore c = u' - \frac{du'}{dx} x'. \; . \; . \; . \; . \; . \; . \; . \; (3)$$

In the immediate vicinity of Q, C will coincide with B only at the point Q, for since C moves uniformly at the same rate at which B passes through Q, if B's rate is increasing, B will from this point move faster than C, and if decreasing slower.

Let C pass over the space $y - c$ while A passes over the space x, since C moves uniformly the relation between x and y must be expressed by an equation of the form
$$y - c = ex. \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

We have evidently
$$\frac{du'}{dx} = \frac{y-c}{x} = \frac{ex}{x} = e.$$

Substituting this value of e, and the value of c from (3) in (4), we get
$$y - u' = \frac{du'}{dx}\left(x - x'\right). \ldots\ldots\ldots\ldots (5)$$

Subtracting (2) from (1), member from member, we have
$$u - u' = a(x^n - x'^n). \ldots\ldots\ldots\ldots (6)$$

Dividing (6) by (5), member by member, we have
$$\frac{u-u'}{y-u'} = a\left(\frac{x^n - x'^n}{x - x'}\right)\frac{dx}{du'};$$

whence
$$\frac{du'}{dx} = a\left(\frac{x^n - x'^n}{x - x'}\right)\left(\frac{y - u'}{u - u'}\right).$$

This equation is true for every corresponding value of u, y and x; but when $x = x'$, $y = u'$, and
$$\left(\frac{y - u'}{u - u'}\right)_{x=x'} = \frac{u' - u'}{u' - u'} = 1;$$

whence
$$\frac{du'}{dx} = a\left(\frac{x^n - x'^n}{x - x'}\right)_{x=x'} = anx'^{n-1}.$$

See Davies' Bourdon p. 257. Dropping the accents, since the solution depends on no condition belonging to the particular values of x' and u' more than to any other corresponding values of x and u, we have
$$du = anx^{n-1}\,dx.$$

Suppose the relation between u and x to be expressed by the equation
$$u = \varphi(x),$$
where $\varphi(x)$ is a symbol representing any algebraic expression involving x. We have
$$\varphi(x) = \varphi[x' + (x - x')].$$
Suppose the second member of this equation to be developed, and arranged according to the ascending powers of $(x - x')$, then
$$\varphi(x) = \varphi[x' + (x - x')] = \varphi(x') + P(x - x') + Q(x - x')^2 + \&c.,$$
where P, Q, &c., are algebraic expressions containing x'. The development must commence with the zero power of $(x - x')$ since it must reduce to $\varphi(x')$ when $x = x'$; whence $u - u' = \varphi(x) - \varphi(x') = P(x - x') + Q(x - x')^2 + \&c.$

In the same manner as before we get

$$\frac{du'}{dx} = \frac{y-u'}{u-u'}\left(\frac{\varphi(x)-\varphi(x')}{x-x'}\right) = \frac{y-u'}{u-u'}\left(P + Q(x-x') + \&c.\right);$$

whence, making $x = x'$, we get

$$\frac{du'}{dx} = \left(\frac{\varphi(x)-\varphi(x')}{x-x'}\right)_{x=x'} = P.$$

NOTE ON THE CORRECTION OF AN ERROR IN THE THEORY OF POLYCONIC PROJECTIONS, BY PROF. W. W. JOHNSON.— In the publications of the Coast Survey* it is erroneously stated that in the Polyconic Projection of the Sphere the projected meridians and parallels cut each other at right angles *throughout the entire area.*

The Polyconic Projection is that in which a central meridian and the equator are represented by straight lines, perpendicular to one another, and correctly subdivided on the same scale; while the parallels are represented by arcs of circles described with radii proportional to the cotangents of the latitudes, such being the radii of the parallels considered as developed each from the tangent cone of which it forms the line of contact with the sphere.

Letting p denote an arc of longitude measured on the equator, $p \cos L$ will denote an arc of the same longitude in latitude L, and as in the projection this arc is measured on a circle whose radius is $\cot L$, its circular measure (which is denoted by θ) is $\theta = p \sin L$.

Adopting the central meridian in the projection as axis of x, and the equator as axis of y, the co-ordinates of the projection of a point in latitude L and longitude p are readily shown to be

$$y = \cot L \sin \theta,$$
$$x = L + \cot L - \cot L \cos \theta.$$

Making p constant these become the co-ordinates of the projected meridian. Now the inclination to the axis of x of a tangent to this curve being denoted by φ, we shall have $\tan \varphi = dy \div dx;$
and if the curve cuts the parallel at right angles we shall have the radius of the parallel coinciding with the tangent, and φ would be the supplement of θ, or $\tan \theta = - dy \div dx.$

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*Coast Survey Report for 1853 page 99, "a projection results in which all intersections of parallels and meridians take place at right-angles," and again, "Over the entire area of this projection all parallels and meridians intersect at right-angles." The same error occurs in Church's Descriptive Geometry, Art. 233, on the Polyconic Projection. "This has the advantage that the representatives of the parallels and meridians are perpendicular to each other, as in space, which is not the case in Flamstead's method."